

Improving Cooling performance of the mechanical resonator with the two-level-system defects

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We study cooling performance of a realistic mechanical resonator containing defects. The normal cooling method through an optomechanical system does not work efficiently due to those defects. We show by employing periodical σ_z pulses, we can eliminate the interaction between defects and their surrounded heat baths up to the first order of time. Compared with the cooling performance of no σ_z pulses case, much better cooling results are obtained. Moreover, this pulse sequence has an ability to improve the cooling performance of the resonator with different defects energy gaps and different defects damping rates.

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I. INTRODUCTION

Preparing a ground state of a mechanical resonator is an important topic, and it has applications in testing fundamental quantum theory, exploring the boundary between classical and quantum regions, and studying precision metrology [1–3]. Considering the inevitable interaction between the resonator and its surrounded heat bath, the state of the resonator is far away from the ground state in equilibrium, so cooling the mechanical resonator to the ground state becomes an urgent task. So far, many proposals have been put forward to cool the resonator. Making use of the capacitive coupling, Lorentz force, magnetic field or strain field induced coupling with a two-level-system (Josephson qubit, or negatively charged nitrogen-vacancy center, etc.), one can cool the mechanical resonator efficiently [4–13]. Other proposals design an optomechanical system consisting of one cavity mode and the resonator to be cooled. The radiative pressure from photons is applied to cool the resonator [14–19]. For all of these proposals, we can finally obtain a resonator with very small phonon number in the long time limit.

In a realistic experimental optomechanical system, the mechanical resonator is often made by silica. Because of the amorphous nature of silicon, defects reside in the amorphous native oxide of the silicon surface [20–23]. As shown in Ref. [24], due to the coupling between the defect with resonator and their couplings with heat baths, the thermal noise from the heat bath of the defects can be effectively transferred to the mechanical resonator. Therefore, the normal cooling method used in the optomechanical system can actually hardly work.

In this paper, we show by employing the periodical σ_z

pulses, we can efficiently remove the detrimental effect of defects and cool the resonator efficiently. The reason is that the periodical σ_z pulses can induce the sign of operators σ_- , σ_+ to be flipped [25–31]. The interaction between defects and the surrounded heat bath can be eliminated up to the first order of time. We display the cooling performance of the resonator with different defects energy gaps and defects damping rates. We find that with a large number of σ_z pulses ($N = 99$), the phonon occupation of the resonator reduces to a lower value, when compared with the case of no σ_z pulses. Besides, we also study the difference of the cooling results through two different calculation approaches, one uses the master equation based on polariton doublets, the other is "simple approach" in which the defects and coupling are added into the master equation of the bare resonator directly [24]. In our discussion, we find that these two approaches give a similar qualitative picture in cooling, but the obtained value of phonon occupations are quantitative different.

The structure of this paper is as follows, Sec. II introduces the total system we shall study. In Sec. III, we explore the cooling performance with different defects energy gaps and different defects damping rates. The paper is concluded by Sec. IV.

II. MODEL

The total system contains one cavity mode, a mechanical resonator, defects and their surrounded heat baths. The total Hamiltonian is [23, 24],

$$H_{tot} = H_{om} + H_{JC} + H_{a,e} + H_{b,e} + H_{\sigma,e} + H_B. \quad (1)$$

Here, H_B denotes the Hamiltonian of the three non-interacting baths. An optomechanical component consists of one cavity mode and the mechanical resonator,

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that is,

$$H_{om} = -\hbar\Delta_L a^\dagger a + \hbar g(a + a^\dagger)(b + b^\dagger), \quad (2)$$

where $\Delta_L = \omega_L - \omega_c$ is the detuning of cavity driving frequency ω_L from the cavity mode frequency ω_c . The operators a (a^\dagger) and b (b^\dagger) stand for the annihilation (creation) operator of the cavity mode and mechanical resonator, respectively.

As discussed above, the defects couple to the mechanical resonator inevitably. Here, we only consider the case of one defect, which can be regarded as a two-level-system (TLS) [20–24]. The Hamiltonian for the resonator and the defect can be written as a Jaynes-Cummings (JC) form [23, 24],

$$H_{JC} = \hbar\omega_m b^\dagger b + \frac{1}{2}\hbar\omega_z \sigma_z + \hbar\lambda(\sigma_+ b + b^\dagger \sigma_-), \quad (3)$$

where ω_z (ω_m) is the frequency of a TLS (mechanical resonator), and λ is the coupling strength.

In our discussion below, we take the interactions between each of these three systems with their surrounding heat bath into account. The interaction Hamiltonian takes the form as,

$$H_{a,e} = \sum_k g_{a,k}(aa_k^\dagger + a^\dagger a_k), \quad (4a)$$

$$H_{b,e} = \sum_k g_{b,k}(bb_k^\dagger + b^\dagger b_k), \quad (4b)$$

$$H_{\sigma,e} = \sum_k g_{\sigma,k}(\sigma_- c_k^\dagger + \sigma_+ c_k). \quad (4c)$$

Here, $g_{a,k}$, $g_{b,k}$, and $g_{\sigma,k}$ are the coupling strength between the cavity mode, the mechanical resonator, the defects and each one's heat bath. The bath modes are labeled by k .

In many cases, the coupling strengths g and λ are comparable, and the cavity damping rate is large. We define polariton states as [24],

$$|n, \alpha\rangle = c_\alpha^n |n \downarrow\rangle + s_\alpha^n |(n-1) \uparrow\rangle, \quad (5)$$

with $n \geq 1$ and $\alpha = \pm$, $|n\rangle$ is the Fock state of the mechanical resonator, $|\uparrow\rangle$ and $|\downarrow\rangle$ are the eigenstates of σ_z . Here, $c_+^n = -s_-^n = \cos(\delta_n/2)$, $s_+^n = c_-^n = \sin(\delta_n/2)$, the expression of δ_n satisfies the relation, $\cos(\delta_n/2) = \sqrt{(\omega_n + \delta\omega)/2\omega_n}$, where $\delta\omega = \omega_m - \omega_z$, and $\omega_n = \sqrt{\delta\omega^2 + 4\lambda^2 n}$.

By employing the projection operator technique [32, 33], and only keeping terms up to the second order of g , we can obtain the dynamics of the reduced density matrix ρ_s for the composite system of the TLS and res-

onator [24],

$$\begin{aligned} \dot{\rho}_s = & -\frac{i}{\hbar}[H_\tau, \rho_s] + \sum_{n,\alpha,\beta} \frac{\Gamma_0^{n\alpha\beta}}{2} \mathcal{L}_0^{n\alpha\beta} \rho_s \\ & + \sum_{n,\alpha,\beta} |A_{\beta,\alpha}^{(n)}|^2 \left[\frac{\Gamma_{-, \alpha\beta}^n}{2} \mathcal{L}(O_n^{\alpha\beta}) + \frac{\Gamma_{+, \alpha\beta}^n}{2} \mathcal{L}(O_n^{\alpha\beta\dagger}) \right] \rho_s, \end{aligned} \quad (6)$$

here, $\mathcal{L}_0^{n\alpha\beta} = (n_{th}^{n\alpha\beta} + 1)\mathcal{L}(O_n^{\alpha\beta}) + n_{th}^{n\alpha\beta}\mathcal{L}(O_n^{\alpha\beta\dagger})$, and $\mathcal{L}(o)\rho = 2o\rho o^\dagger - \rho o^\dagger o - o^\dagger o\rho$. The polariton Hamiltonian $H_\tau = \omega_{n,\alpha}|n, \alpha\rangle\langle n, \alpha|$, with $\omega_{n,\alpha}$ is the eigenenergies of the polariton states. The thermal population $n_{th}^{n\alpha\beta} = (\exp(\hbar\omega_{n\alpha\beta}/k_B T) - 1)^{-1}$, with $\omega_{n\alpha\beta} = \omega_{n,\alpha} - \omega_{n-1,\beta}$. Here, all heat baths have the same temperature T . The operator $O_n^{\alpha\beta} = |(n-1), \beta\rangle\langle n, \alpha|$. The expressions of $\Gamma_0^{n\alpha\beta}$ and $\Gamma_{\mp, \alpha\beta}^n$ are,

$$\Gamma_0^{n\alpha\beta} = |A_{\beta,\alpha}^{(n)}|^2 \gamma_m + |\sigma_{\beta,\alpha}^{(n)}|^2 \gamma_\tau, \quad (7a)$$

$$\Gamma_{\mp, \alpha\beta}^n = \frac{g^2 \kappa}{\kappa^2/4 + (\omega_{n\alpha\beta} \pm \Delta_b)^2}, \quad (7b)$$

where γ_m , γ_τ and κ stand for the damping rate of the mechanical resonator, the TLS, and the cavity mode, respectively. The coefficients $A_{\beta,\alpha}^{(n)}$ and $\sigma_{\beta,\alpha}^{(n)}$ are from the expressions of $b = \sum A_{\beta,\alpha}^{(n)} O_n^{\alpha\beta}$, $\sigma_- = \sum \sigma_{\beta,\alpha}^{(n)} O_n^{\alpha\beta}$.

σ_z pulses. Many researches have been devoted to designing different pulse sequences, to keep the system away from decoherence induced by the surrounded heat bath [25–31]. Here, we use the periodical σ_z pulses, to eliminate the interaction between the TLS and the bath up to the first order of time. Considering the properties of the operator σ_z ,

$$\sigma_z \sigma_- \sigma_z = -\sigma_-, \quad \sigma_z \sigma_+ \sigma_z = -\sigma_+, \quad (8)$$

we can obtain,

$$\begin{aligned} & \sigma_z e^{-iH_{tot}t} \sigma_z e^{-iH_{tot}t} \\ & = e^{-it\sigma_z H_{tot}} \sigma_z e^{-iH_{tot}t} = e^{-itf_1 + itf_2} \cdot e^{-itf_1 - itf_2}, \end{aligned} \quad (9)$$

with the coefficients $f_1 = H_{om} + \hbar\omega_m b^\dagger b + \frac{1}{2}\hbar\omega_z \sigma_z + H_{a,e} + H_{b,e} + H_B$, $f_2 = \hbar\lambda(\sigma_+ b + b^\dagger \sigma_-) + H_{\sigma,e}$. Clearly, up to the first order of time, we can eliminate the interaction of the TLS and the corresponding heat bath. Combining Eq. (6) and (9), we obtain the time evolution for the density matrix of $\rho_s(t)$.

$$\begin{aligned} \rho_s(t) \Rightarrow \\ \dots e^{\mathcal{H}(t_j - t_{j-1})} \sigma_z \dots \sigma_z \{ e^{\mathcal{H}(t_2 - t_1)} \sigma_z \{ e^{\mathcal{H}t_1} \rho_s \} \sigma_z \} \sigma_z \dots \sigma_z \dots \end{aligned} \quad (10)$$

Here, $e^{\mathcal{H}(t_k - t_{k-1})} \rho_s$ denotes the system evolution obeying Eq. (6), $t_k - t_{k-1}$ is the time duration between two

adjacent pulses. Within these, we can now investigate the cooling performance of the resonator numerically.

III. RESULTS AND DISCUSSIONS

Firstly, by using the master equation based on polariton doublets, we discuss the cooling performance of the mechanical resonator with different number of σ_z pulses, see Fig. 1. (a). The frequency of the mechanical resonator is set as $\omega_m = 200$ MHz. As a result, the more σ_z pulses are used, the lower the resonator phonon number is. If the number of σ_z pulses is too small ($N = 9$), the

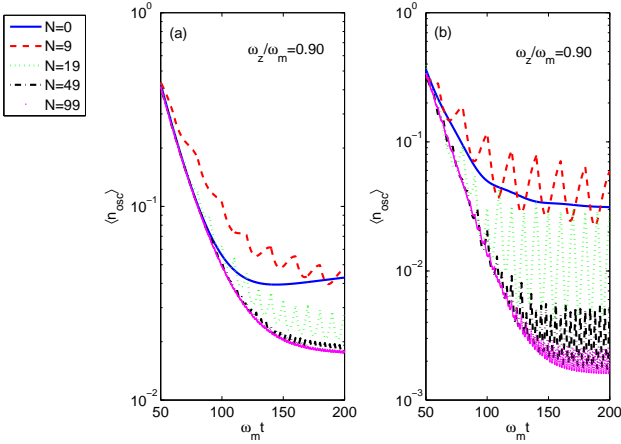


FIG. 1. (Color online): Residual phonon number ($\langle n_{osc} \rangle$) versus different number of σ_z pulses. (a) master equation based on polariton doublets. (b) "simple approach". Parameters set: $\omega_z/\omega_m = 0.9$, $\kappa/\omega_m = 0.15$, $\gamma_m/\omega_m = 10^{-6}$, $\gamma_\tau/\omega_m = 2.5 \times 10^{-4}$, $g/\omega_m = 0.05$, $\lambda/\omega_m = 0.05$, $\Delta_L/\omega_m = -1$, $T = 0.1K$.

result is even worse than the case of no σ_z pulses ($N = 0$). Although based on the analysis above, the periodical σ_z pulses can eliminate the interaction between TLS and its surrounding bath up to the first order of time, when the time interval between two adjacent σ_z pulses is too long, the consequence of higher order terms are significant. When there are more σ_z pulses, the time interval between two adjacent incident pulses is smaller, and the effect of those higher order terms becomes less significant. As shown in Fig. 1. (a), we achieve good cooling performance with the pulse numbers $N = 19, 49, 99$. For the case of no σ_z pulses, at time $\omega_m t = 200$, the residual phonon number of the resonator is 0.04289, while for the case of $N = 99$, the residual phonon number of the resonator is 0.01793 at time $\omega_m t = 200$. There is a decrease of 58.2%. In Fig. 1. (b), the cooling results by using the "simple approach" is presented. Same initial states are chosen for both two approaches (Fig. 1. (a) and (b)). We find that the evolution of the population of the resonator given by "simple approach" is qualitatively same as the

case shown in Fig. 1. (a), but the residual phonon number of the resonator in the long-time limit through these two approaches are quantitatively different. As pointed out in Ref. [24], results from the master equation based on polariton doublets (Fig. 1. (a)) are more accurate. From Fig. 1. (a), it is clearly seen that we can cool the resonator more efficiently with more injecting pulse number.

Secondly, we study the cooling performance of the resonator with different defect energy gaps. Fig. 2 presents the results of three different cases ($\omega_z/\omega_m = 0.60, 0.80, 0.95$). Cooling results from different pulse numbers,

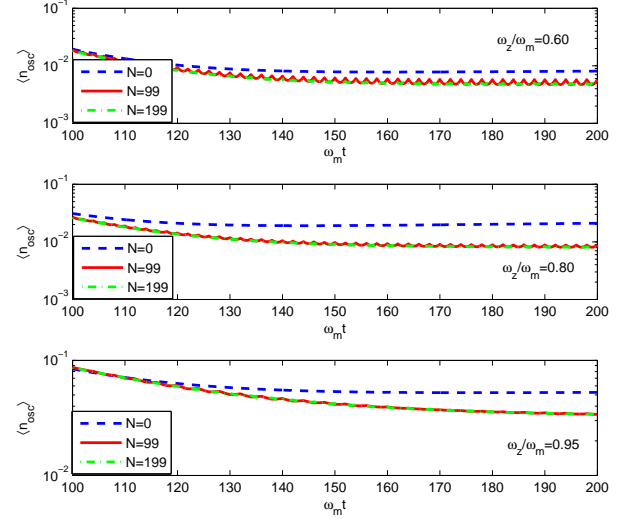


FIG. 2. (Color online): Residual phonon number ($\langle n_{osc} \rangle$) versus different number of σ_z pulses ($N=0, 99, 199$). Parameters set: $\kappa/\omega_m = 0.15$, $\gamma_m/\omega_m = 10^{-6}$, $\gamma_\tau/\omega_m = 2.5 \times 10^{-4}$, $g/\omega_m = 0.05$, $\lambda/\omega_m = 0.05$, $\Delta_L/\omega_m = -1$, $T = 0.1K$. From top to bottom, ω_z/ω_m is 0.6, 0.8, and 0.95.

$N = 99$ and $N = 199$, are compared. These results show, by employing the periodical σ_z pulses, we can cool the resonator efficiently within a wide range of defect energy gaps.

Finally, we compare the cooling performance of the resonator with different TLS damping rates γ_τ in Fig. 3. The near resonance condition is chosen ($\delta\omega/\omega_m = 0.05$), and the parameters satisfy, $\delta\omega \leq \lambda < \kappa$. From Fig. 3. (a), we find that when the TLS damping rate is small enough (between 10^{-6} and 10^{-5}), the cooling performance of the resonator is insensitive to the pulse number applied and the residual phonon number of the resonator is very small ($\simeq 0.004$). In Fig. 3. (b), we change the defect energy gap to a new value, $\omega_z/\omega_m = 0.60$, and the resonance condition is not satisfied, $\delta\omega > \lambda = 0.05\omega_m$. When the TLS damping rate is larger than 5×10^{-5} , the resonator is cooled efficiently with the pulse number $N = 99$, while the result is not effective with no σ_z pulses applied.

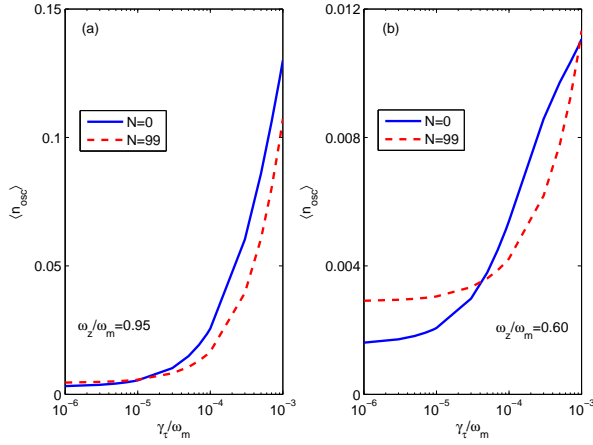


FIG. 3. (Color online): Residual phonon number ($\langle n_{osc} \rangle$) versus γ_τ . Different number of σ_z pulses ($N=0, 99$) are applied. The value of $\langle n_{osc} \rangle$ is chosen at time $\omega_m t = 200$, where the residual phonon number has been stable already. Parameters set: $\kappa/\omega_m = 0.15$, $\gamma_m/\omega_m = 10^{-6}$, $g/\omega_m = 0.05$, $\lambda/\omega_m = 0.05$, $\Delta_L/\omega_m = -1$, $T = 0.1K$.

IV. CONCLUSION

In summary, we introduce periodical σ_z pulses to eliminate the bad effect from the defects in cooling the mechanical resonator. The periodical σ_z pulses can remove the interaction between the TLS and the heat bath up to the first order of time. By applying σ_z pulses, we can cool the resonator efficiently with different defect energy gaps and different TLS damping rates. Other designed pulse sequences eliminating the interaction more than first order of time [26, 28–31] might be more efficient to cool the resonator. This deserves further study in the future.

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